Local Operations in Fault-tolerant Quantum Computation: Gauge Color Codes

Héctor Bombín

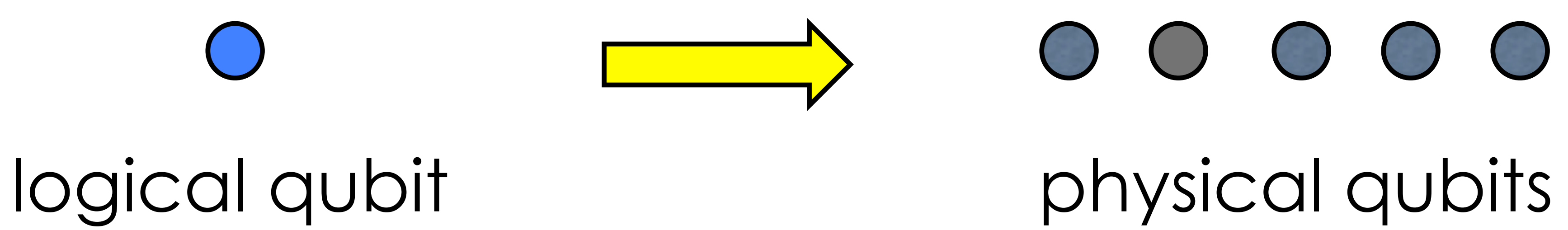
outline

- introduction
    - locality in fault-tolerant quantum comp.
    - topological codes & local operations
- results
- single-shot error correction
    - self-correction
    - gauge color codes
- universality via gauge-fixing

error correction

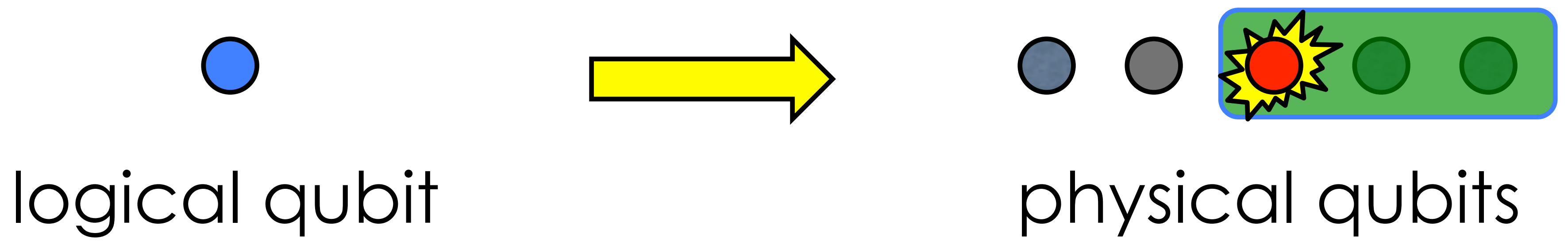
For quantum computation…

- want: isolation + control
- have: decoherence + imprecision
- need: error correction
- how: one qubit encoded in many



error correction

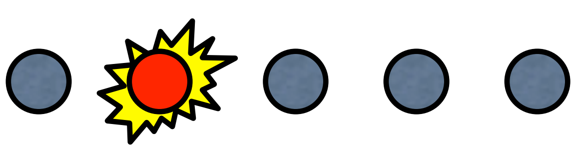
- extra degrees of freedom detect errors
- check operators fix the code subspace
- measuring them gives the error syndrome
- to correct, guess error from syndrome



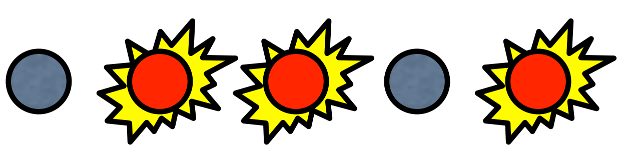
locality

- correction is possible if errors are *not* arbitrary
- local errors are more likely
- phenomenology: local stochastic noise

$$P(\text{error affects qubits } i_1, i_2, \ldots, i_n) \;\leq\; \varepsilon^n$$



more likely



less likely

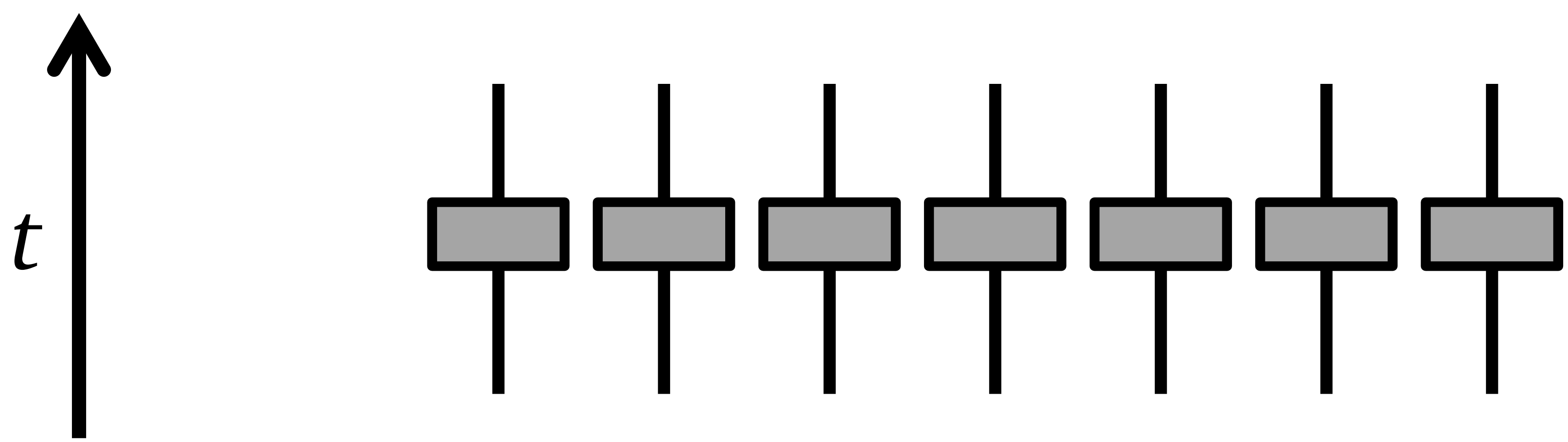
fault-tolerant QC

- compute with encoded qubits
- errors pile up, but error correction flushes them away (up to a point)
- logical operations should preserve locality!

$\varepsilon_0 < \varepsilon_1 < \varepsilon_2 \qquad \varepsilon_0$

transversal operations

- act separately on physical subsystems
- do not spread errors
- downside: never universal Eastin & Knill '09



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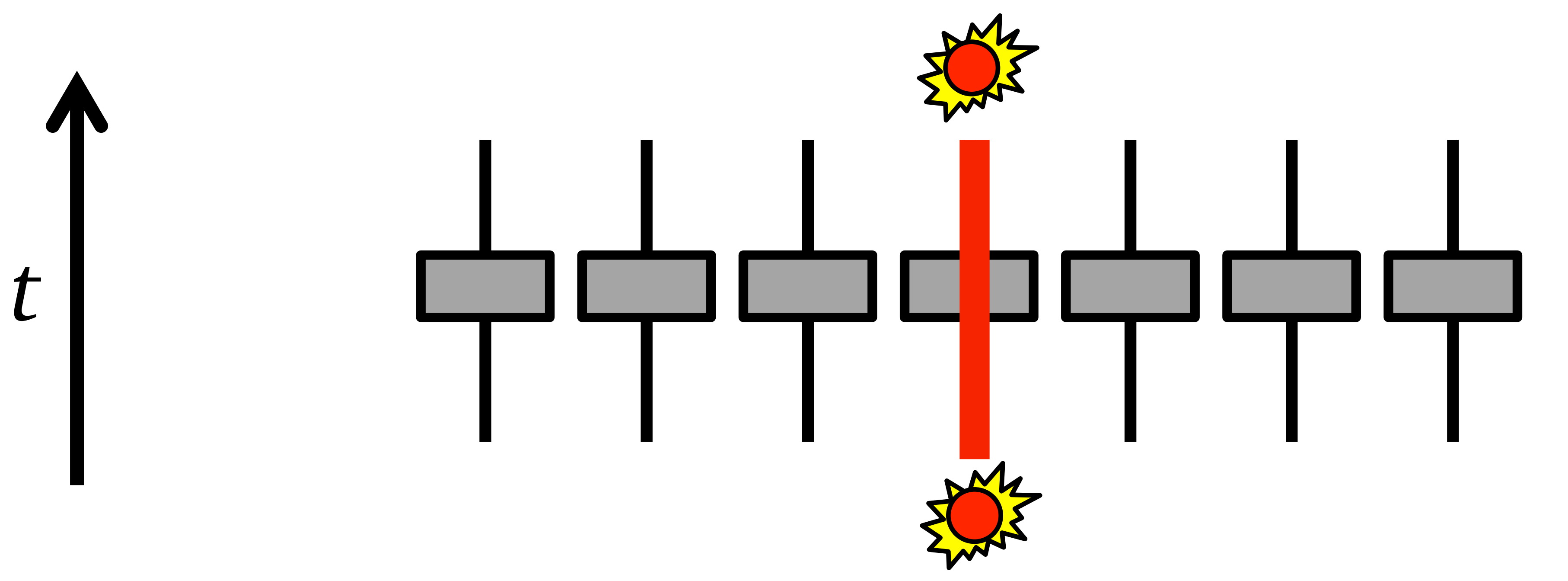
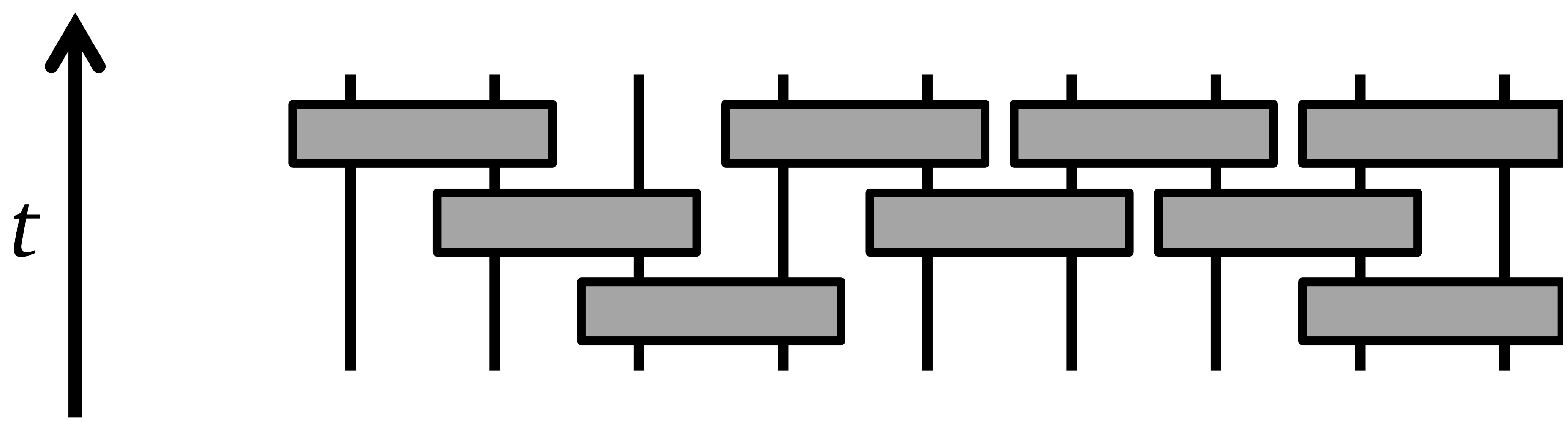


local operations

- finite depth circuit
- limited spread of errors
- in some contexts, limited power Bravyi & König '09,…



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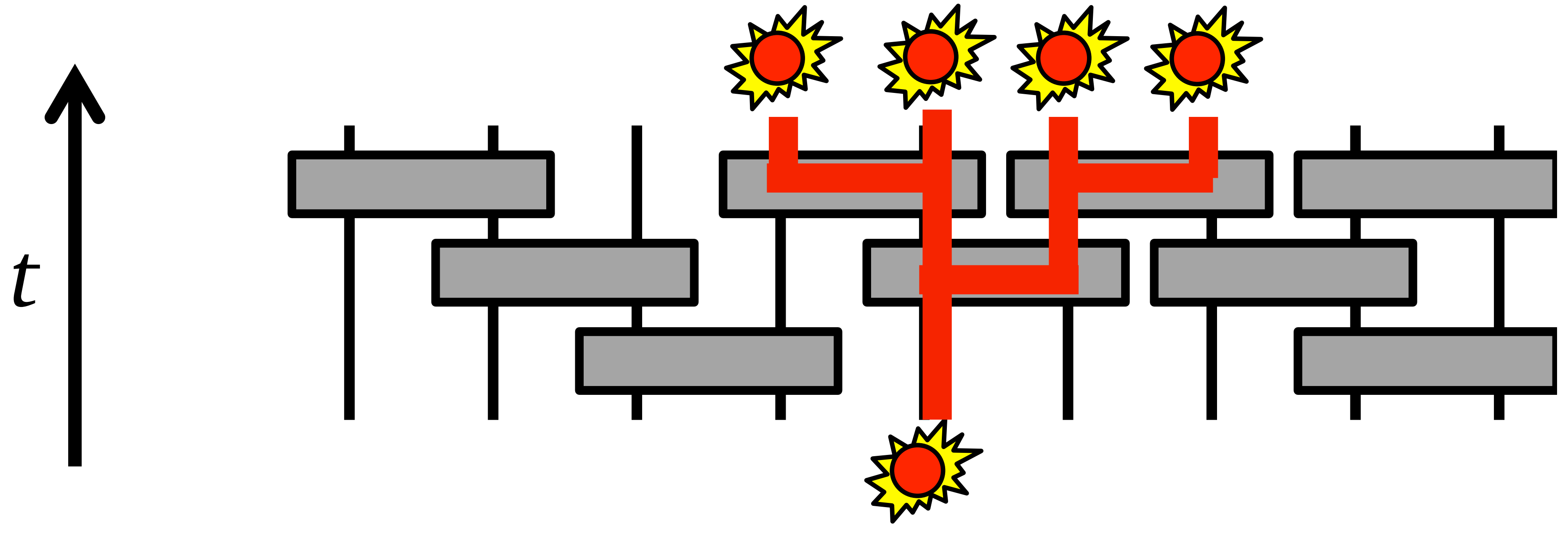
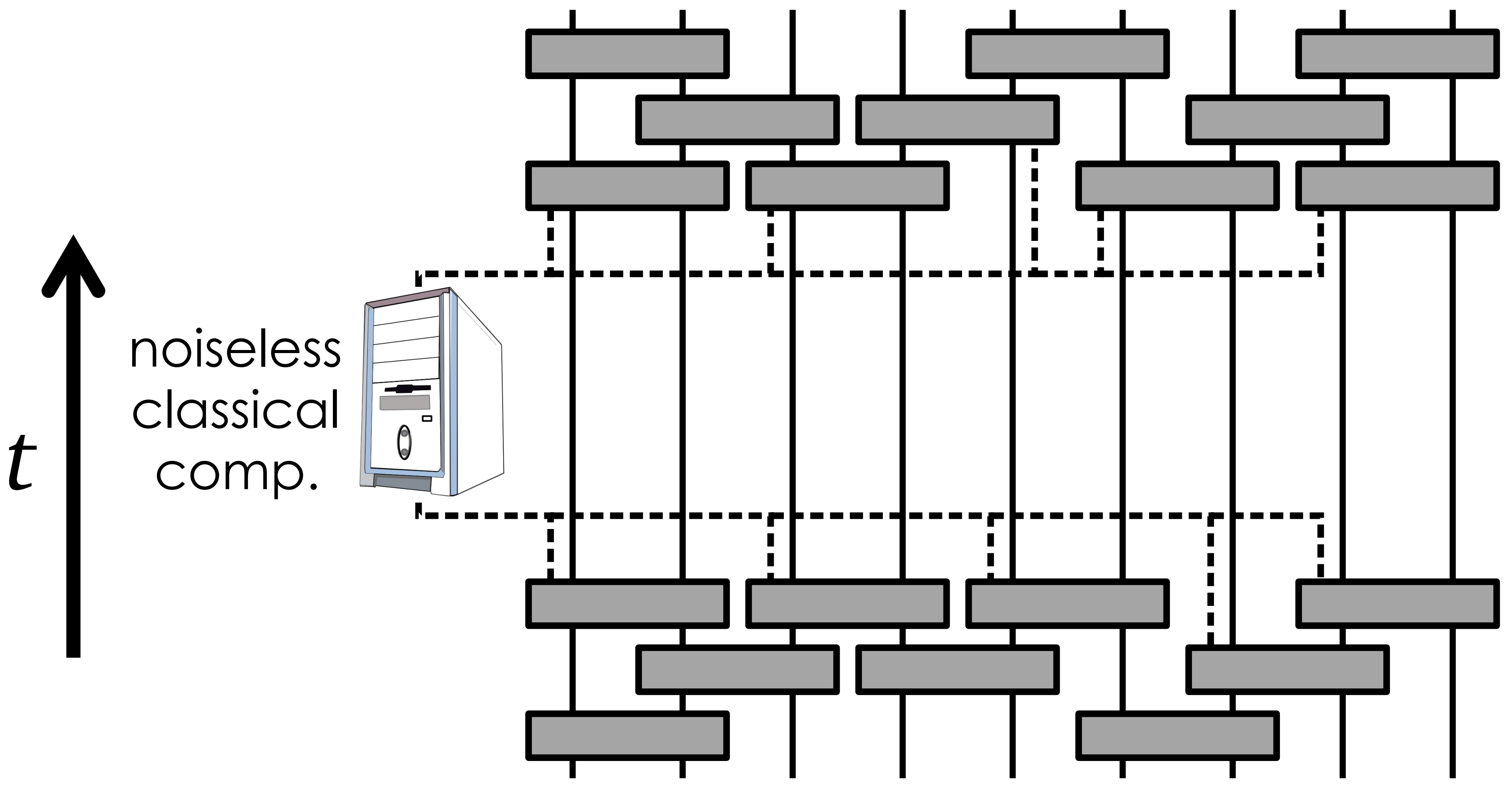


quantum-local operations

- finite depth circuit + global classical comp.
- universal operations + error correction **no limits!**



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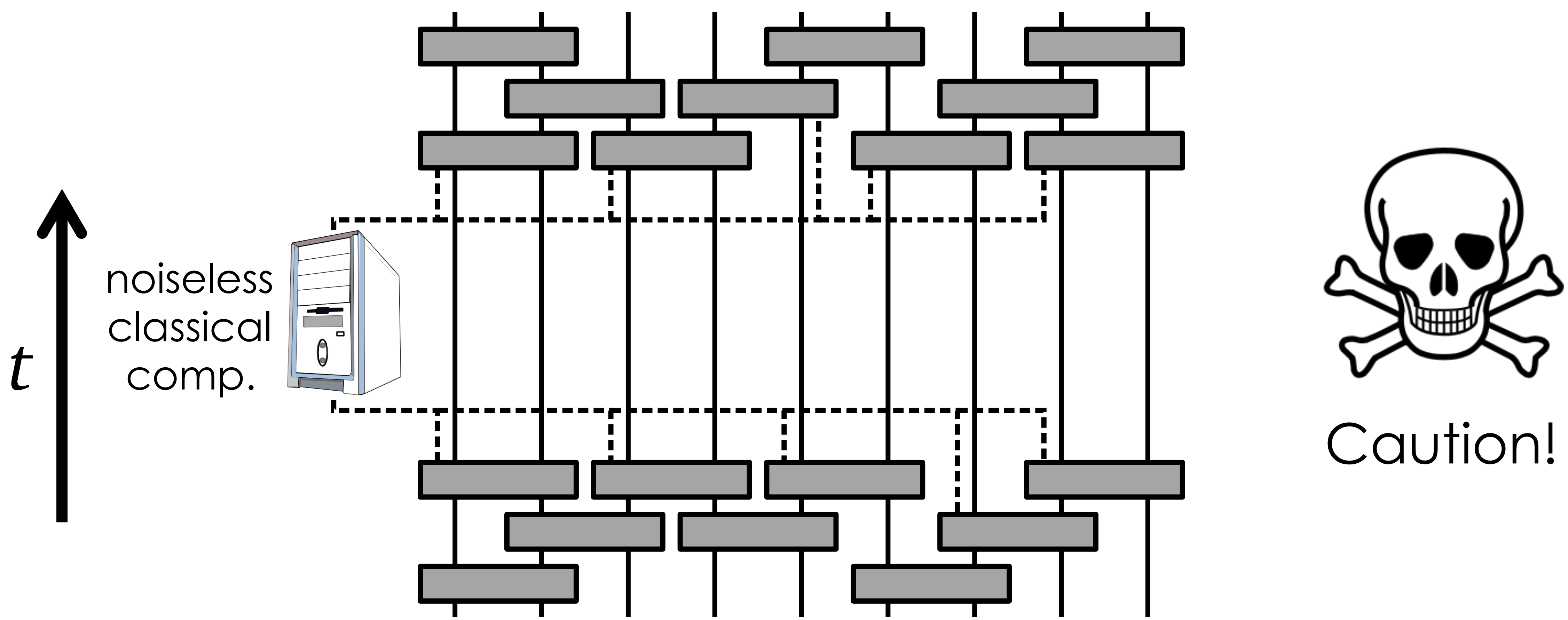

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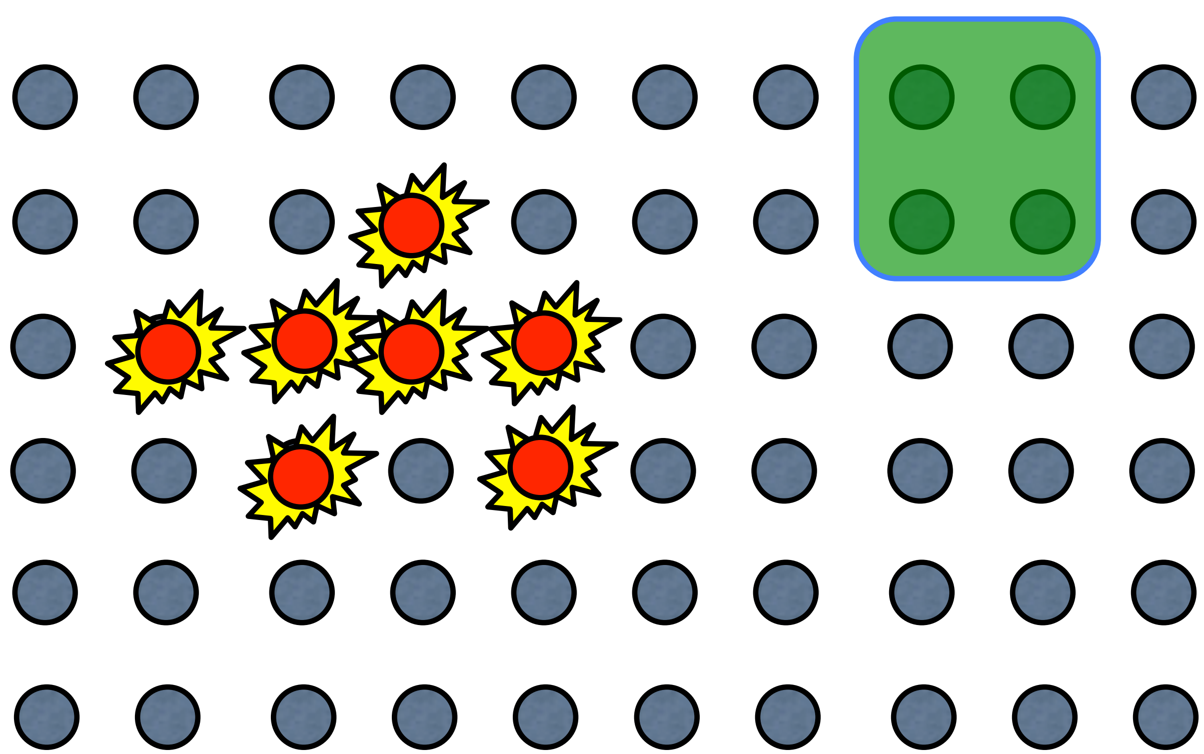
topological codes

Kitaev '97

- physical qubits on a lattice
- local check operators
- 'local' operators cannot harm logical qubits



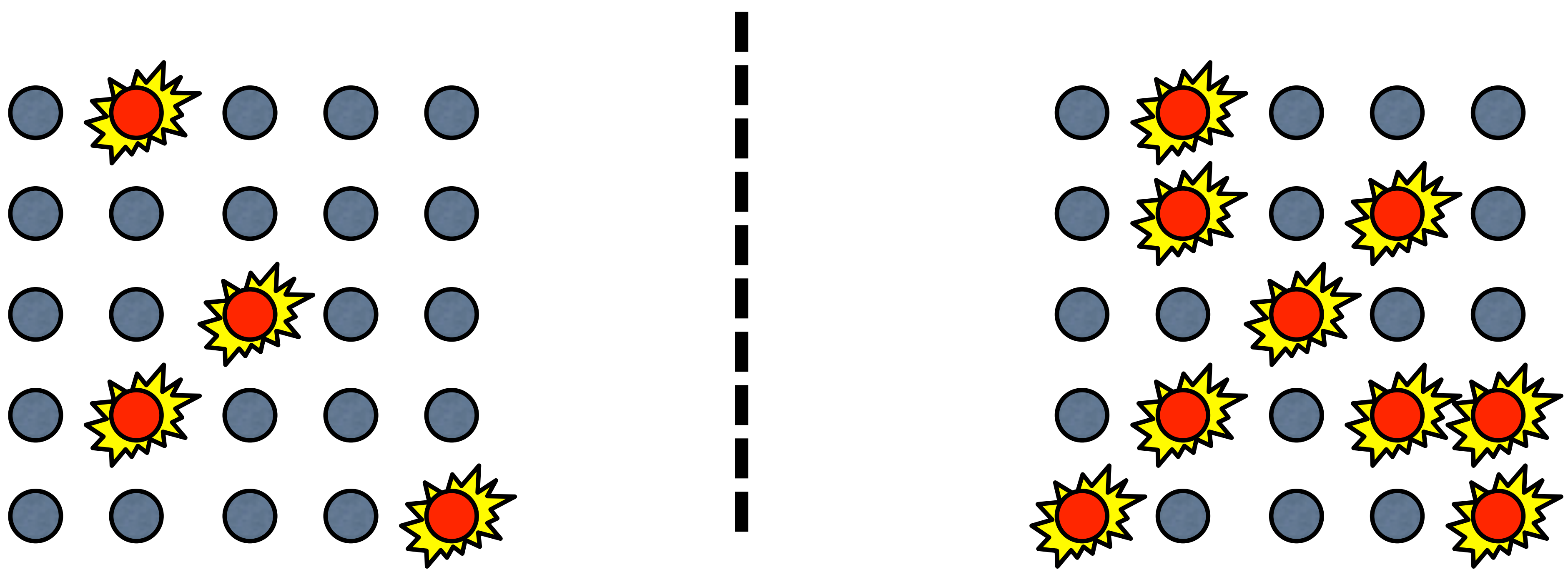
topological codes

Kitaev '97

low ε

error threshold / phase transition

high ε



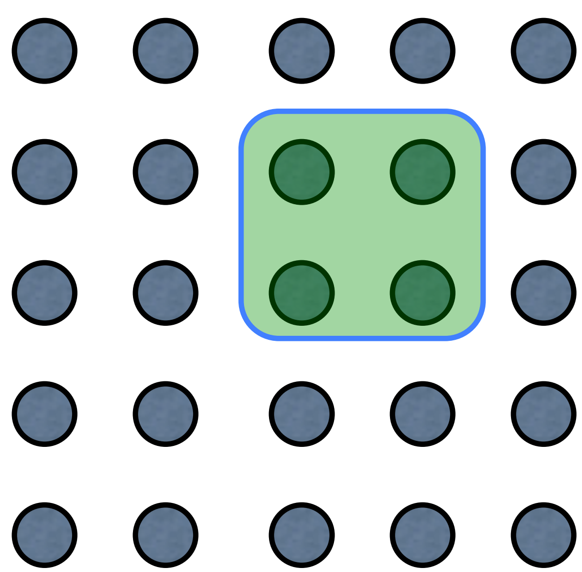
perfect correction

large systems

information destroyed

topological order

- gapped (local) quantum Hamiltonian
- locally undistinguishable ground states
- robust against deformations



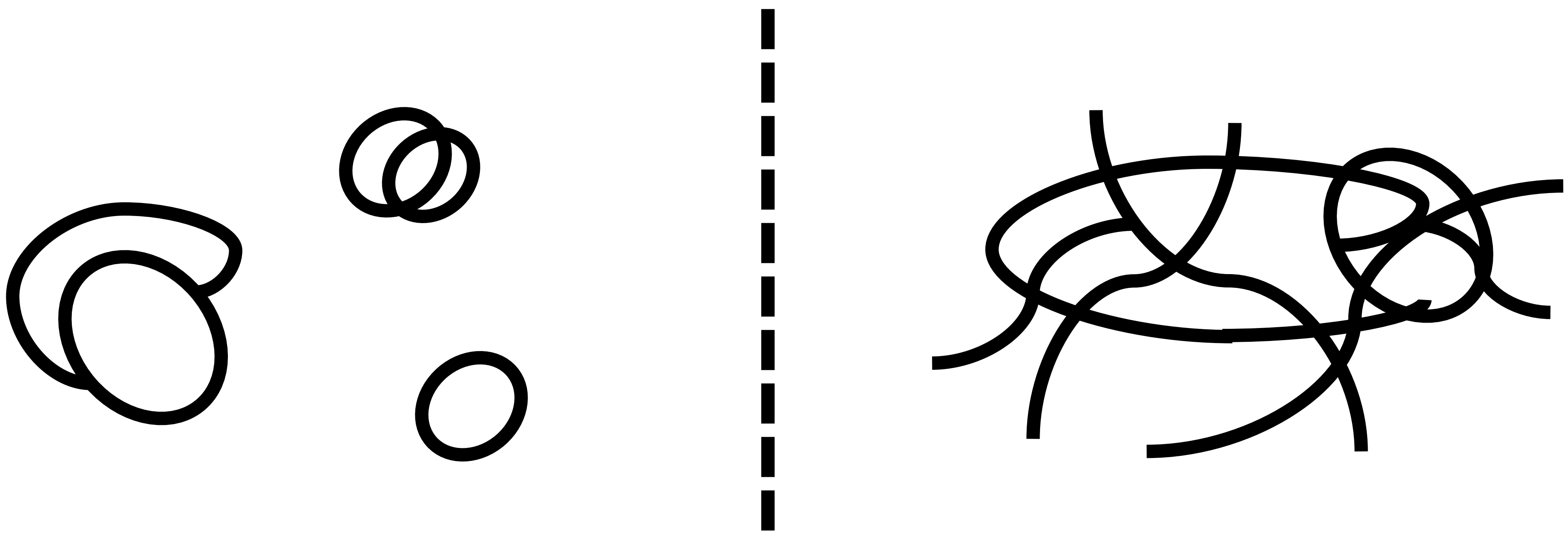
$$H = -J \sum_i P_i$$

self-correction

Dennis *et al* '02

- for D ≥ 4 excitations can be extended objects

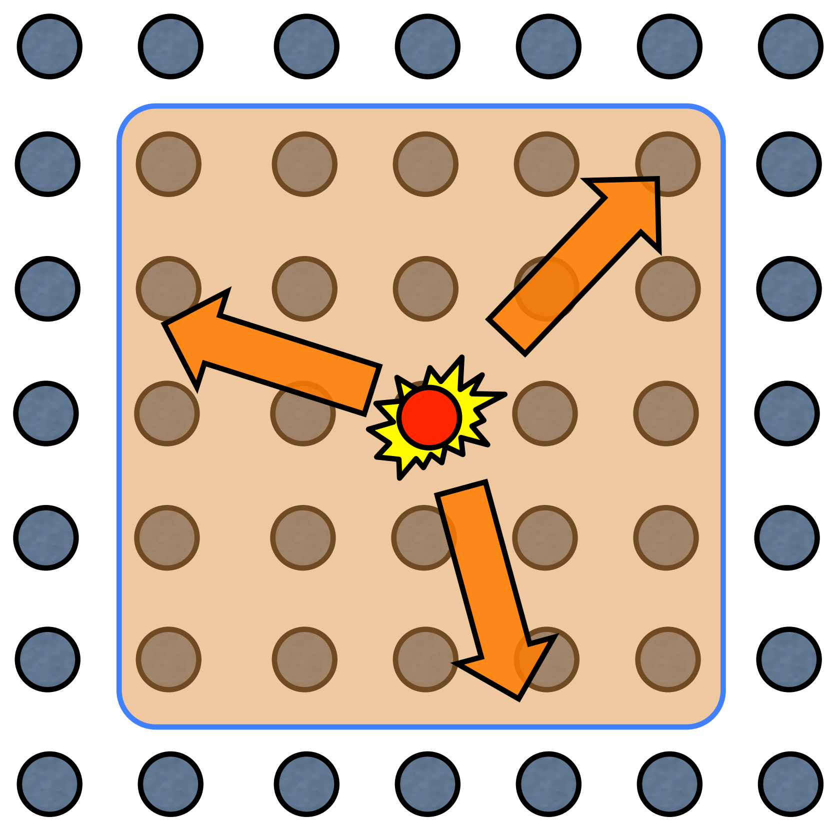
low T - **confined** T_C high T - **unconfined**



perfect protection ⟵ large systems ⟶ information destroyed

local operations

- *geometrically* local, finite depth circuit
- finite spatial spread of errors

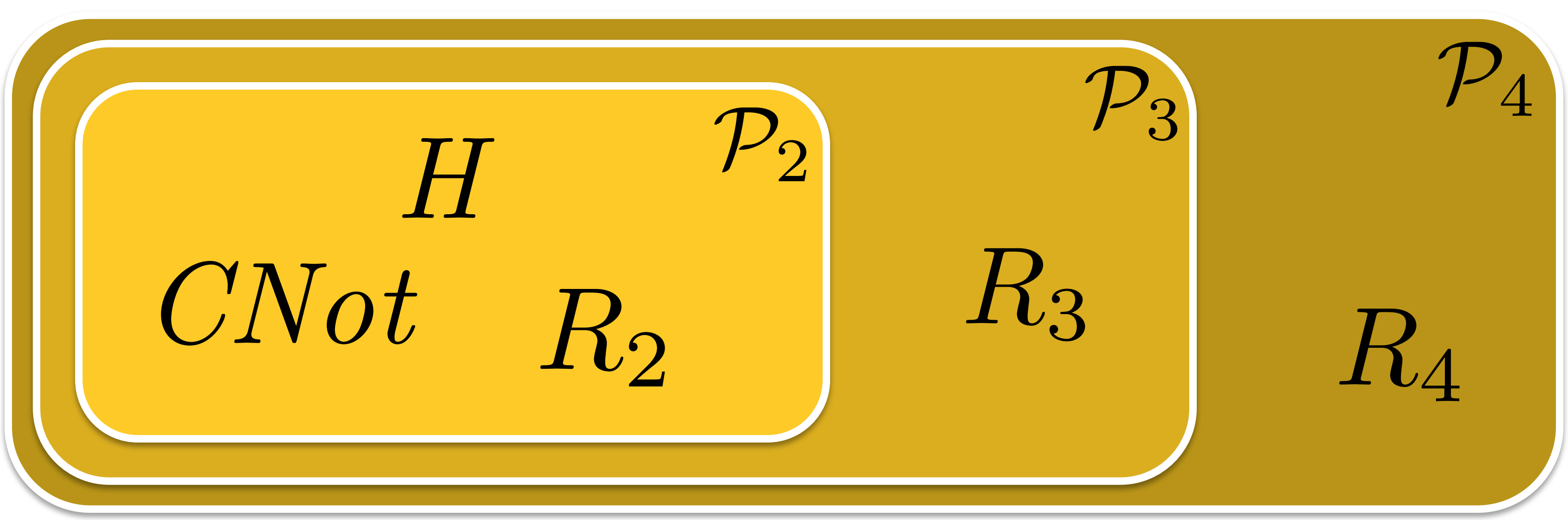


Dimensional restrictions

Bravyi & König '13

- top. stabilizer codes: check ops in Pauli group
- geometrical constraints on local gates

$$\mathcal{P}_D := \{U \,|\, U\mathcal{P}U^\dagger \subseteq \mathcal{P}_{D-1}\}, \qquad \mathcal{P}_1 := \mathcal{P}$$



$$R_D := \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^D} \end{pmatrix}$$

...

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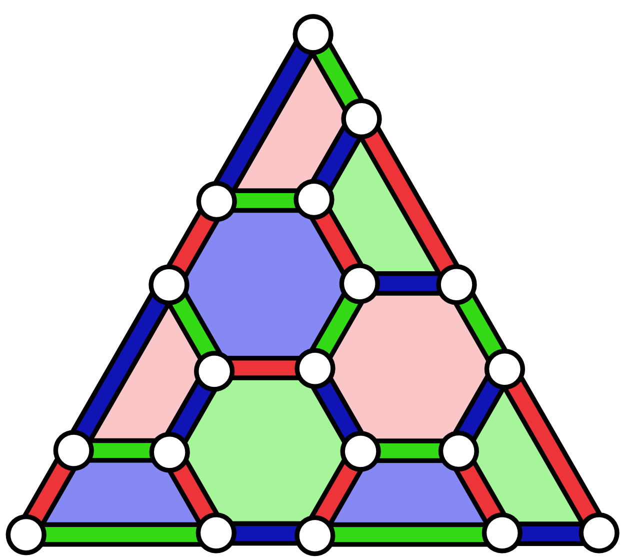
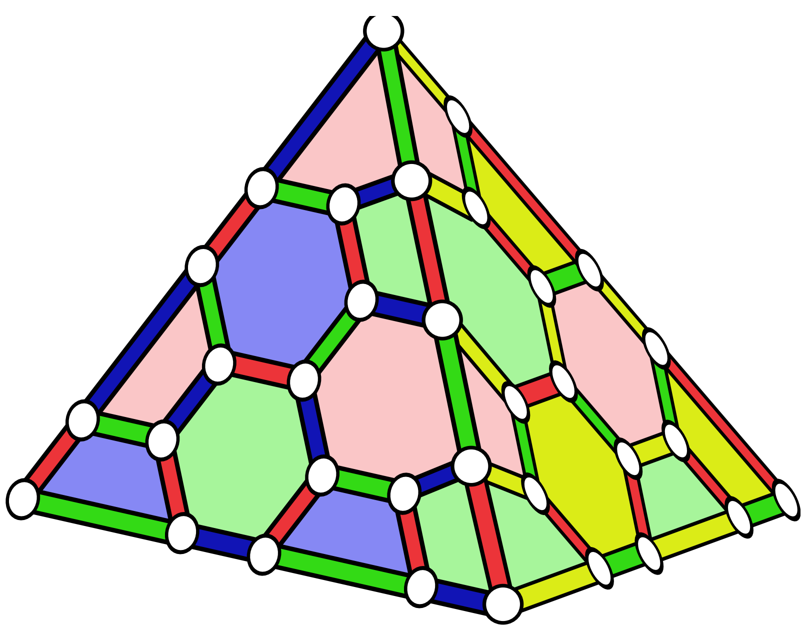
color codes

- topological stabilizer codes defined for any D
- optimal transversal gates: R_D transversal



Clifford group



CNOT + T

arXiv:1311.0879

subsystem codes

Poulin '05

- *gauge* (free) degrees of freedom
- in topological codes, can be local
- more local measurements
- **gauge fixing**: gauge ops $\rightarrow$ check ops

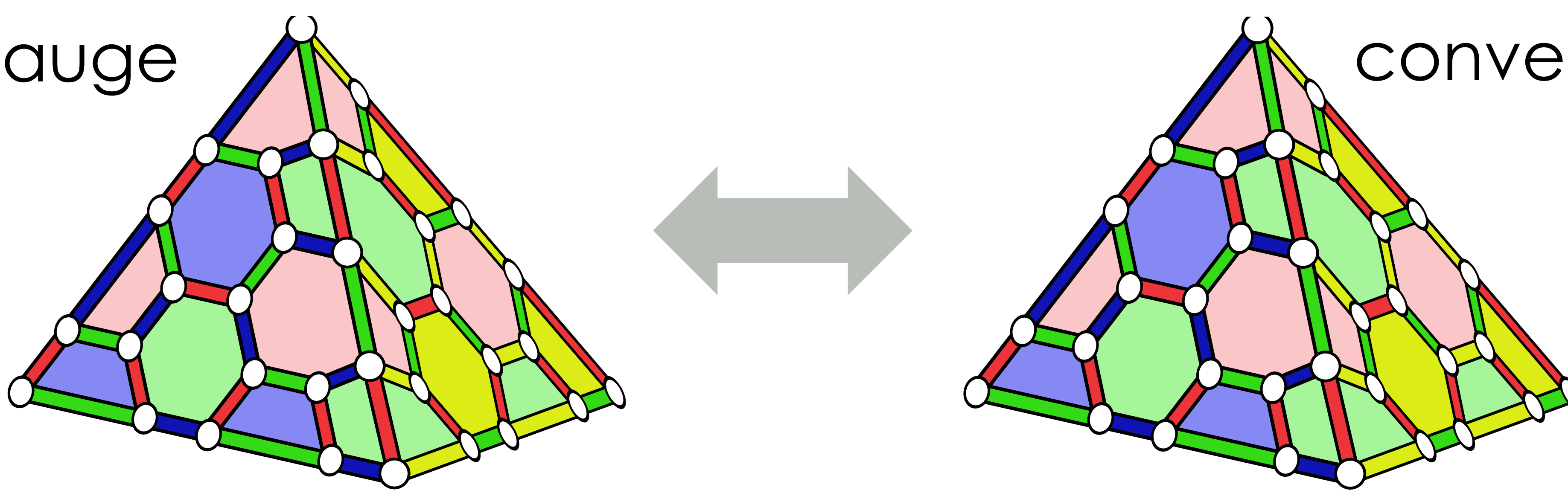
  Paetznick & Reichardt '13
- amounts to error correction
- allows to combine properties of codes
  (e.g. transversal gates for universality)

3D gauge color codes

- 6-local measurements, as in 2D
- universal transversal gates via gauge fixing

gauge

conventional



CNOT + H

CNOT + T

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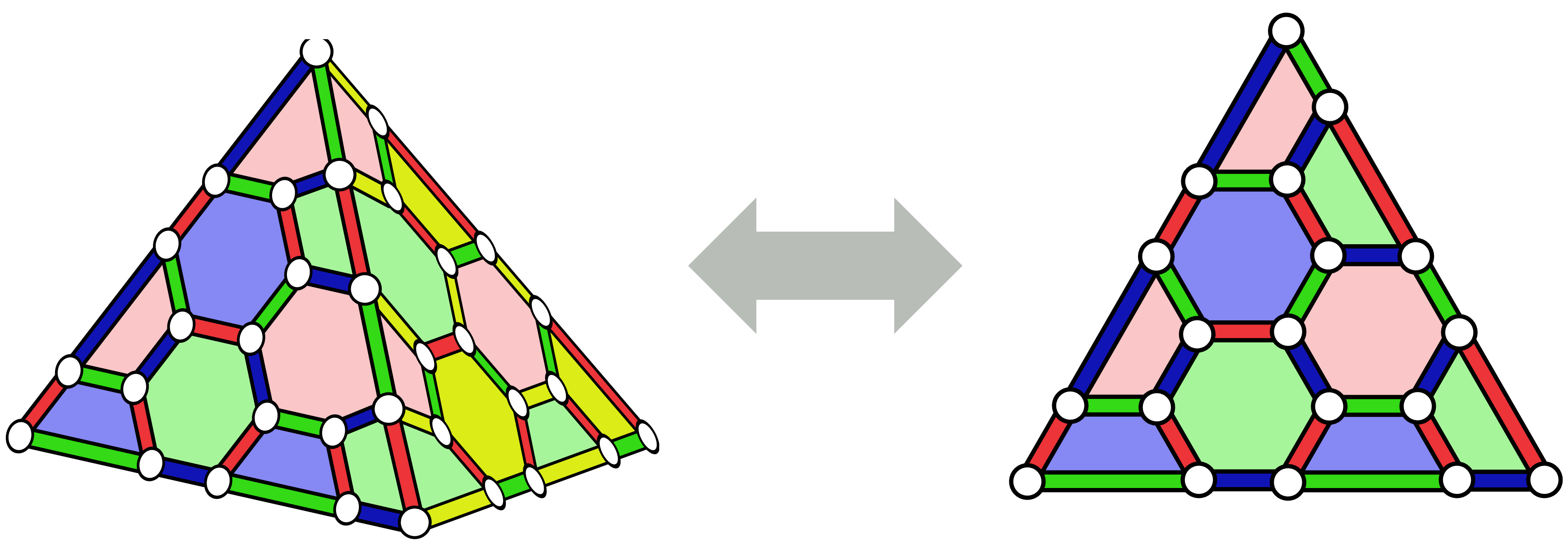
3D gauge color codes

bonus!

- ***dimensional jumps*** via gauge fixing
- 2D color codes require much less qubits



arXiv:1412.5079

quantum-local error correction

- in topological stabilizer codes ***ideal*** error correction is q-local
- but real measurements are ***noisy***, and multiple rounds are required (to avoid large errors)

quantum-local error correction

- some codes are inherently robust!
- local measurement errors yield local errors
- **single-shot error correction** (no multiple rounds)
- linked to self-correction: confinement

arXiv:1404.5504

quantum-local error correction

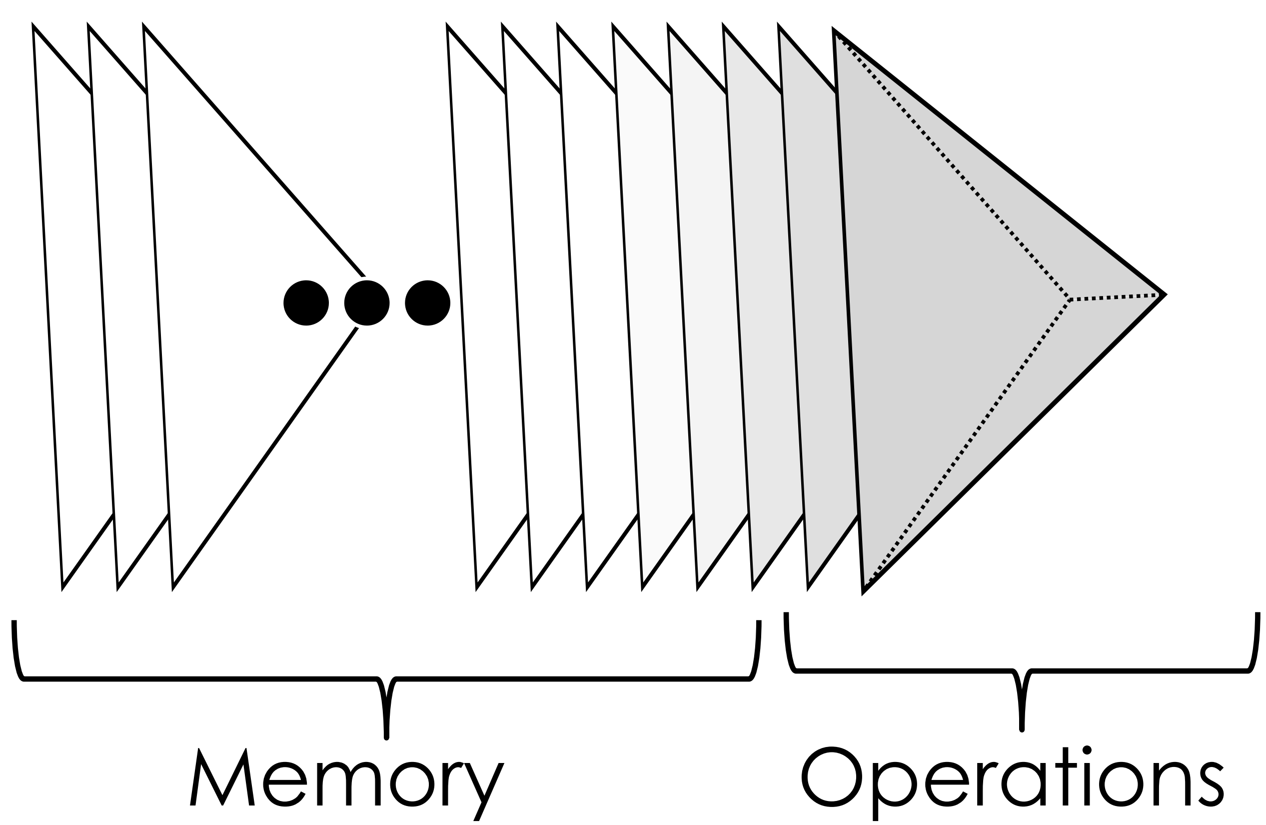
- 3D gauge color codes *are* single-shot!
- *confinement* due to gauge ‘redundancy’
- also single-shot gauge-fixing

arXiv:1404.5504

3D-local constant time QC

bonus!

- fault-tolerant QC in 3D qubit lattice
- local quantum ops + global classical comp.
- constant time ops. (disregarding *efficient* CC)



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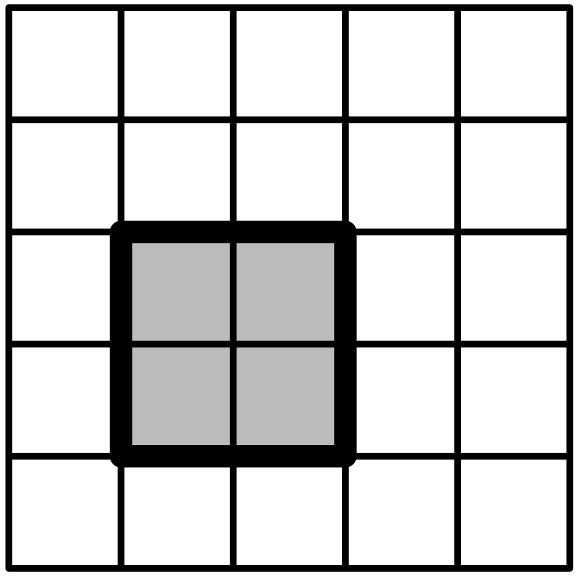
Ising model

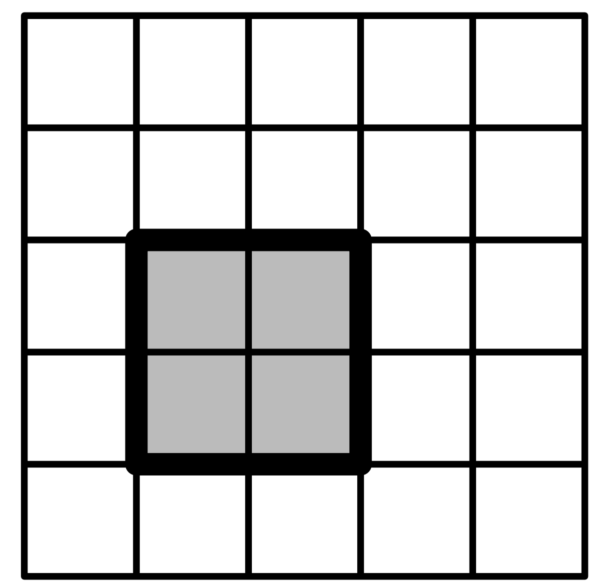
- simplest (classical) self-correction
- critical temperature T_C if $D>1$
- below T_C → **confined** loops
- stable bit (exponential lifetime)



repetition code *à la* Ising

- stabilizer code for bit-flip errors
- qubits = faces
- check operators = edges

$$Z_e := Z_i Z_j$$

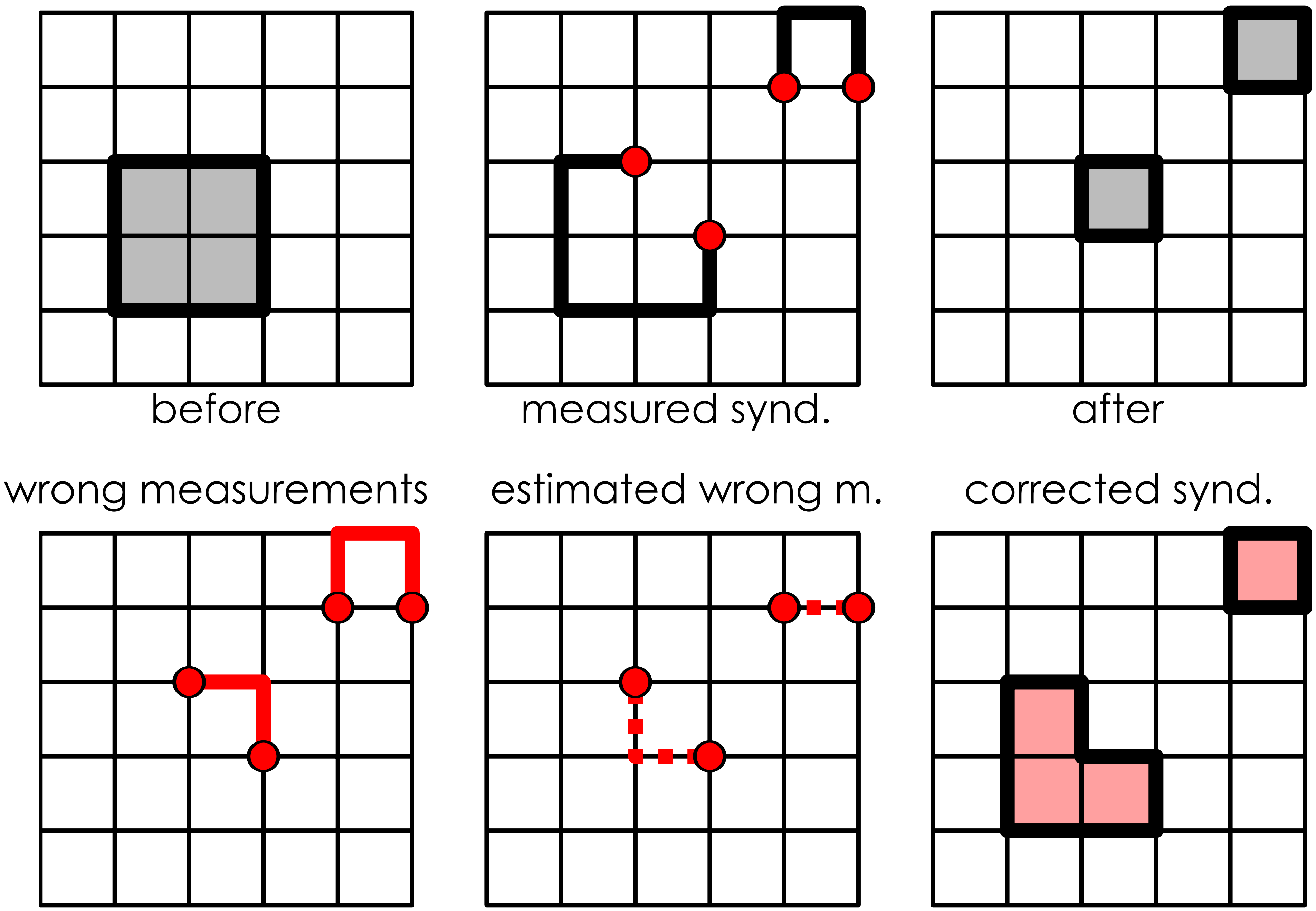


- syndrome composed of loops
- low local noise ⟶ **confined** loops

noisy error correction

- assume noisy measurements *only*
- goal: confined residual loops

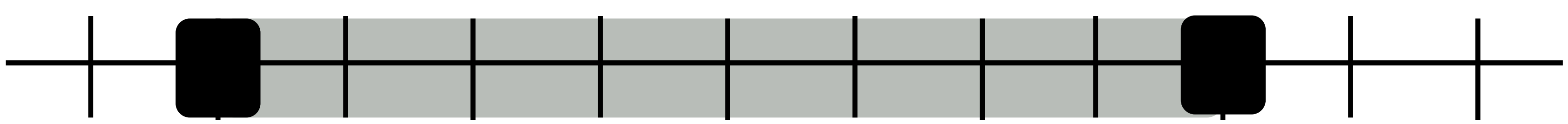
noisy error correction



effective wrong measurements = residual syndrome

spatial dimension

- 1D Ising / repetition code:
  unconfined excitations / syndrome



- confinement mechanism: extended excitations
- full quantum self-correction seems to require $D>3$

Dennis *et al* '02

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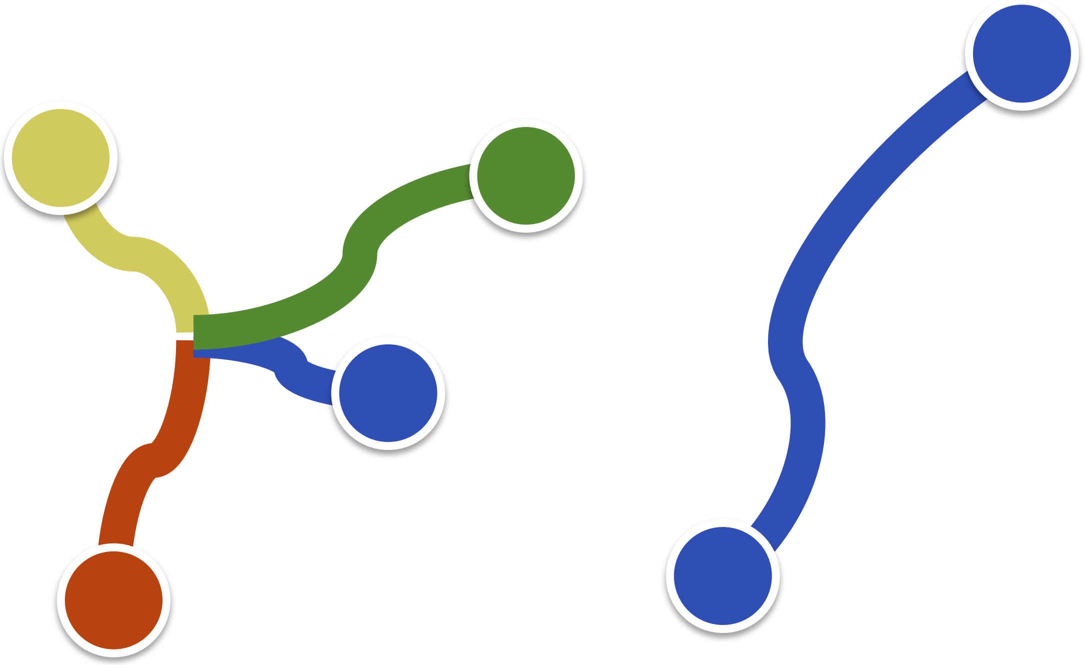
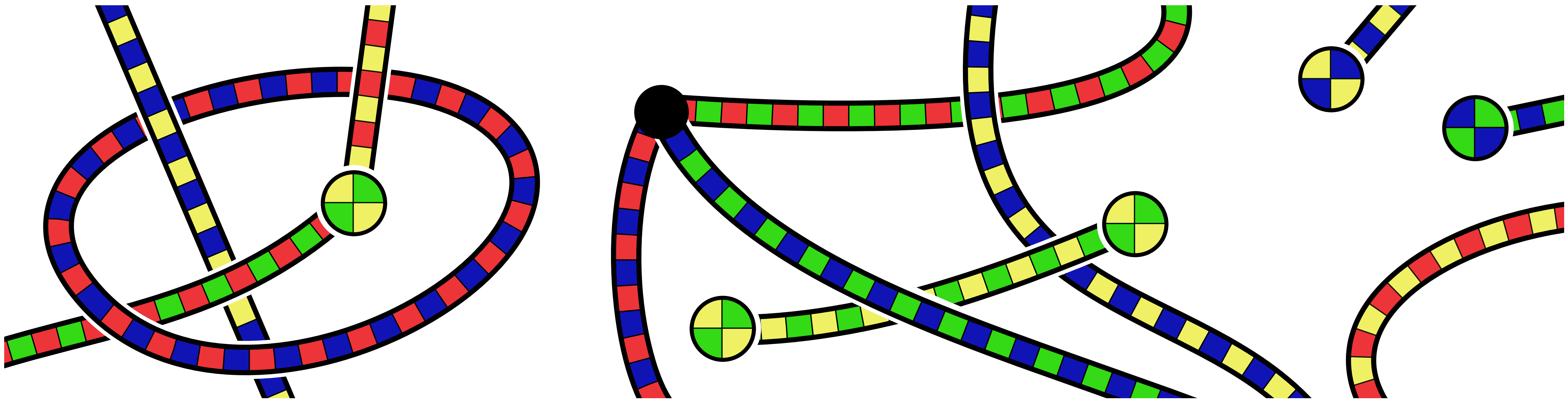
confinement in 3D

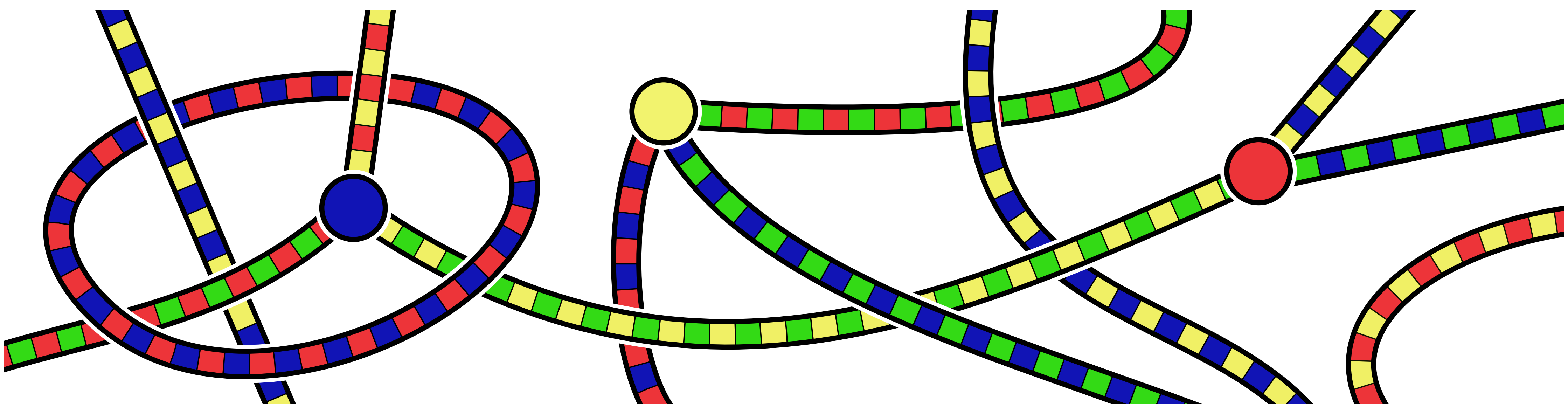


- 3D gauge color codes:
- errors: string-net like
- syndrome: endpoints
- conserved color charge
- direct measurement of syndrome: no confinement
- instead, obtain it from gauge syndrome
- another application of subsystem codes!

confinement in 3D

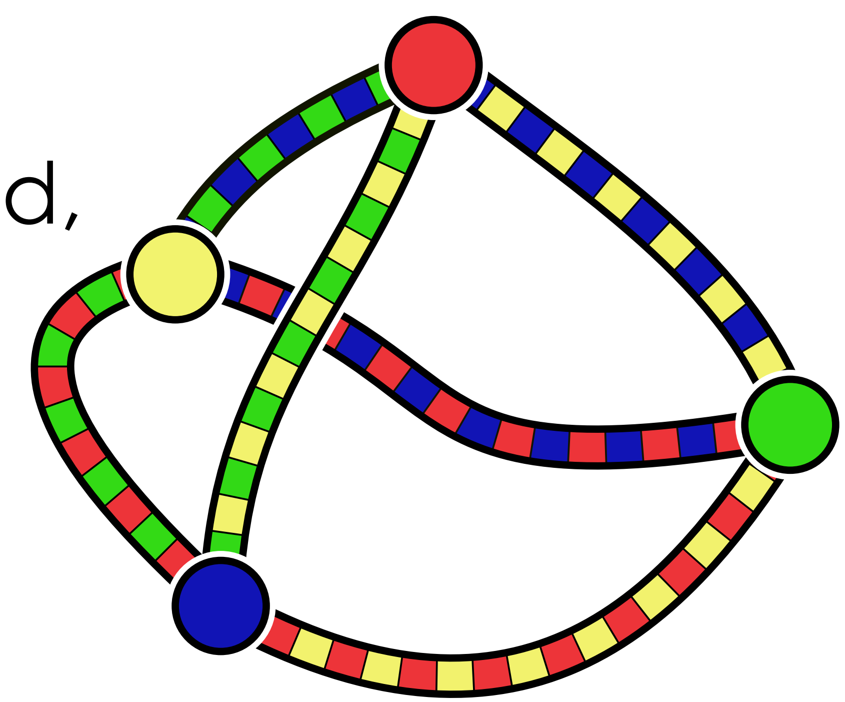


faulty gauge syndrome: endpoints = syndrome of faults



repaired gauge syndrome: branching points = syndrome

confinement in 3D



- the gauge syndrome is unconfined, it is *random* except for the fixed branching points
- the (effective) wrong part of the gauge syndrome *is* confined
- each connected component has branching points with neutral charge (i.e. locally correctable).

- branching points exhibit charge confinement!

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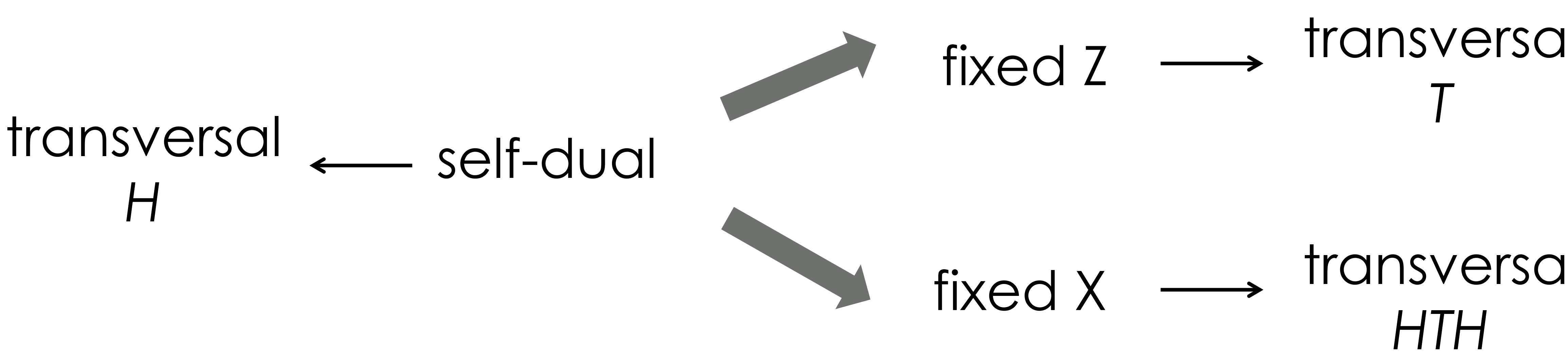
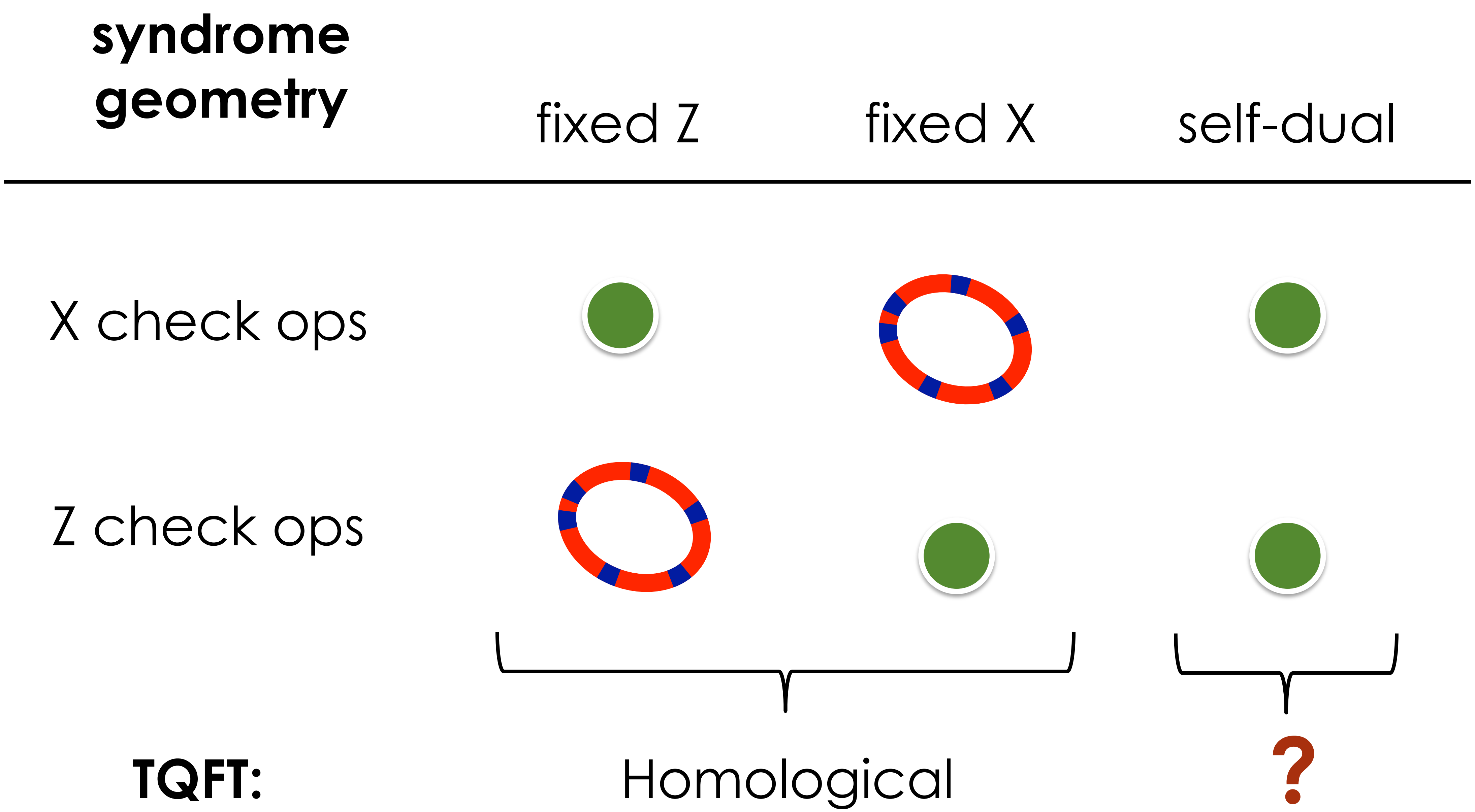
gauge fixing

- there is an X and a Z gauge syndrome
- any of them can be fixed to become part of the stabilizer, but not both!
- each option corresponds to a conventional 3D color code



gauge fixing

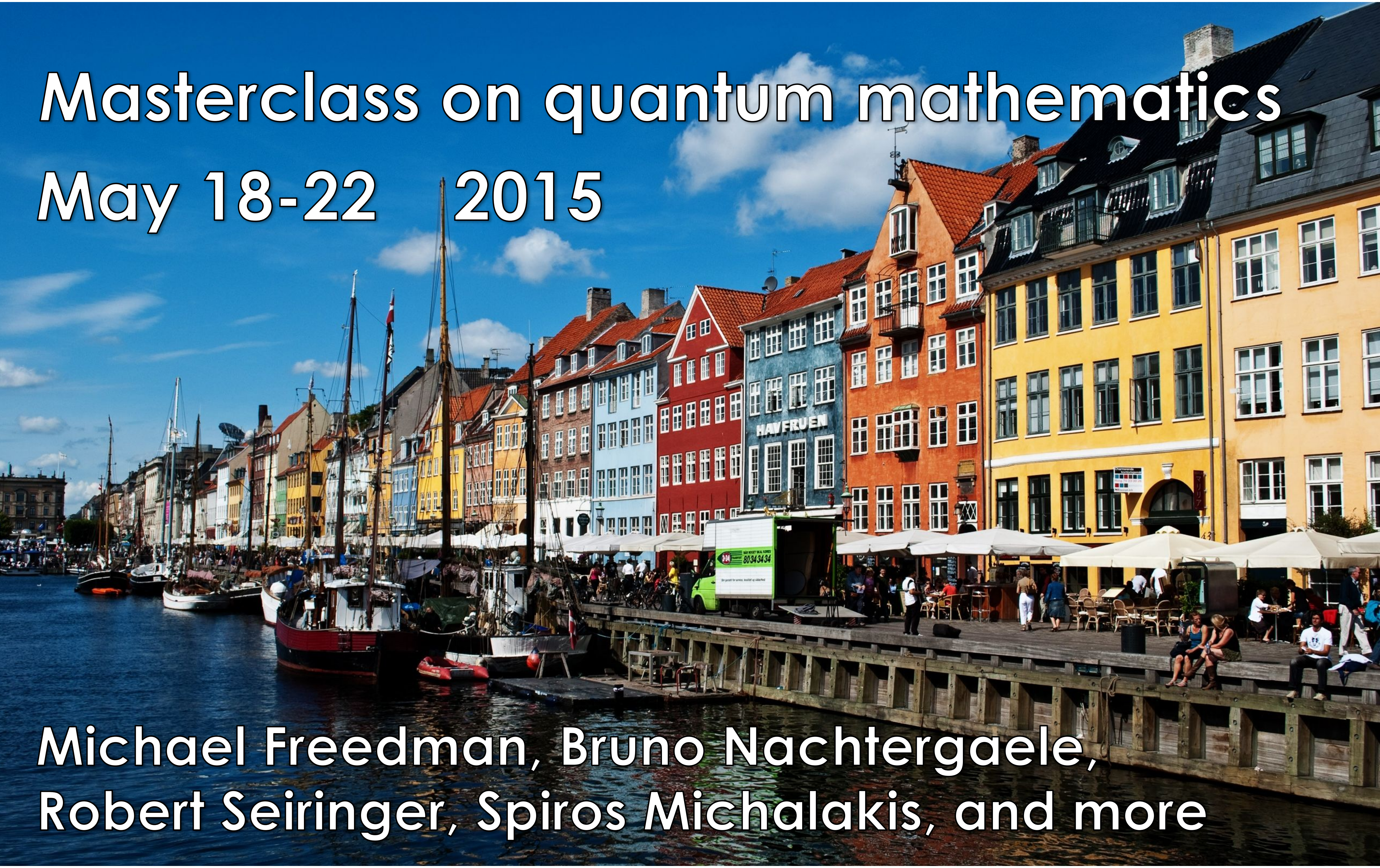


summary & future work

- color codes have optimal transversal gates
- universality via gauge fixing
- single-shot error correction is possible and is linked to self-correction
- 3D-local FTQC with constant time overhead
- what are the limitations in 2D?
- what about non-geometrical locality?
- related 3D self-correcting systems?

New QI group in COPENHAGEN!

Wanted:
Phd students & postdocs



http://www.math.ku.dk/english/research/conferences/2015/qmath-masterclass/